Internet Appendix:

Managing Liquidity in Production Networks:
The Role of Central Firms

IA 1 A Stylized Model

IA 1.1 Setup

The model develops through two periods: At $t = 0$, each firm starts with some initial wealth W_0 and decides how much to distribute P and how much to keep inside the firm as cash holdings C_0. At $t = 1$, Firm 1 (2) receives a random, independent cash inflow $W_{1,s}$ ($W_{2,s}$) depending on the state of the world $s \in S$. The realization of the profit at $t = 1$ constitutes a liquidity shock to the firm, who also faces the opportunity to invest in projects. Firm 1 (2) can borrow l_s (b_s) from an outside investor (bank). Firm 1 (2) also faces an investment opportunity with productivity μ_s (η_s) and decides to invest k_s (I_s).

I assume that each firm's production function depends on the production level of the other firm. Specifically, firm 1's production function is $\Phi_1(I_s)^{1-\alpha}k^\alpha = (m_1 + \phi_1 I_s)^{1-\alpha}k^\alpha$. In other words, $\mu_s = (m_1+\phi_1 I_s)^{1-\alpha}$. Similarly, I assume that firm 2's production function is $\Phi_2(k_s)^{1-\alpha}I^\alpha = (m_2+\phi_2 k_s)^{1-\alpha}I^\alpha$, i.e., $\eta_s = (m_2 + \phi_2 k_s)^{1-\alpha}$. Here, ϕ_i indicates firm i's connectivity ($i = 1, 2$; ϕ_1 may not equal ϕ_2) and m_i is a firm-specific constant. Higher values of ϕ_i indicate that firm i is more affected by the other firm, thus more connected.

At $t = 2$, firms pay back their loans with the profit from their projects and consume the rest. Firms choose the level of investment, financing, and cash holdings to maximize expected payout (i.e., firm value). The funding market is competitive with borrowing cost R, while the cost of inter-firm financing r_s is endogenously determined. Cash holdings inside the firm are subject to a proportional cost δ.

Firm 1 and firm 2 face symmetric choices. I only model Firm 1's optimization problem below, while Firm 2's problem is symmetric:

$$EV = \max_{C_0,k_s,C_{1,s},l_s,i_s} E_s[V_{1,s}] + P \tag{1}$$

$$s.t.\ V_{1,s} = k_s^\alpha \mu_s + C_{1,s}(1-\delta) - l_s R - i_s r_s \tag{2}$$

$$k_s + C_{1,s} = W_{1,s} + C_0(1-\delta) + l_s + i_s \qquad (\lambda_{1,s}) \tag{3}$$

$$P + C_0 = W_0 \qquad (\lambda_0) \tag{4}$$

$$k_s, l_{1,s},\ P,\ C_{1,s},\ C_0 \geq 0, \tag{5}$$

where E_s stands for expectations over $t = 1$ state s; $W_{1,s}$ is the realization of time 1 liquidity for firm 1 at state s; r_s is the inter-firm financing rate at $t = 1$, which is endogenous to firms' investment and financing decisions; $\lambda_{1,s}$ is the Lagrangian multiplier for time 1, state s budget constraint. Eq. (3) is the $t = 1$ budget constraint: firm 1's investment and cash holdings add up to the its total wealth, which equals its proceeds from the projects, previous period cash holdings, and borrowing from the bank or from firm 2. Note that firm 1 has two channels of financing: bank financing and inter-firm financing. For inter-firm financing, the borrower incurs not only the borrowing rate but also the loss of productivity from reducing its counterparty (i.e., the lender)'s investment. The other firm, however, may be willing to accept a lower borrowing rate since it benefits from increased investment by the borrower.

IA 1.2 Solution

At $t = 1$, firms choose investment, financing, and cash holdings at the beginning of each period to optimize their expected equity value. Each firm has two channels of financing: bank financing and inter-firm financing. It is of interest to this study to discuss the conditions under which firms will conduct inter-firm financing, as such conditions will reveal the boundaries where firms will choose whether to borrow, and which avenue to borrow from. To start, I denote $\lambda_{i,s}$ as the marginal value of corporate liquidity for firm i at state s. Firms' decisions satisfy the following first-order conditions:

$$\lambda_{1,s} = (1 - \alpha) k_s^{\alpha} \phi_1 \Phi_1 (I_s)^{-\alpha} + r_s \tag{6}$$

$$\lambda_{2,s} = (1 - \alpha) I_s^{\alpha} \phi_2 \Phi_2 (k_s)^{-\alpha} + r_s \tag{7}$$

where r_s is the interest charged for inter-firm loans. It is endogenously determined by equating the total amount of liquidity in the two firms to their investment expenses: $k_s + I_s \leq W_s = W_{1,s} + W_{2,s}$. The term $(1 - \alpha) k_s^{\alpha} \phi_1 \Phi_1 (I_s)^{-\alpha}$ reflects the opportunity cost of borrowing from a counterparty: by lending to firm 1, firm 2 loses the potential to invest more in its own project. This results in a decline in the profits of firm 1 due to the connection between them, which factors as a cost of obtaining liquidity from inter-firm financing. On the other hand, firm 2 also benefits from lending to firm 1: as firm 1 increases its investment, firm 2 enjoys an increase in its productivity factor, represented by $(1 - \alpha) I_s^{\alpha} \phi_2 \Phi_2 (k_s)^{-\alpha}$.

IA 1.2.1 Investment

The first-order condition for i_s suggests that a firm will borrow from its counterparty if its marginal value of liquidity is lower than bank borrowing rate R. It will otherwise borrow from the bank. The realization of states s at $t = 1$ can thus be divided into three states, the bank-financing state, the inter-firm financing state, and the unconstrained state.

In the bank-borrowing state, W_s is low, and $\lambda_1 = R$ or $\lambda_2 = R$. At least one firm borrows from the bank, and its marginal value of liquidity equals the bank rate. Firms' investments stay at a constant low level $\{\underline{k}, \underline{I}\}$. When two firms have different values of liquidity, the firm with the higher value will borrow from the bank and in turn lend to the other firm. In other words, inter-firm financing can exist alongside bank financing.

In the inter-firm financing state, W_s is still low but both firms' internal values of liquidity drop below the bank lending rate: $1-\delta < \lambda_{i,s} < R,\ i = 1, 2$. In this stage, firms finance each other, and the market-clearing condition $k_s + I_s \leq W_s$ is binding. Firms use all their liquidity for investments, and their investments increase with W_s. Any additional internal liquidity can benefit both firms because of relationship-specific synergy between firms.

The unconstrained state features high realization of liquidity W_s with $\lambda_1 = 1-\delta$ or $\lambda_2 = 1-\delta$, leading to the slack budget constraint $k_s + I_s < W_s$. In this stage, at least one firm will hold cash since the marginal value of investment is low. When both firms' marginal value of liquidity equals $1-\delta$, their investments reach an unconstrained optimal level $\{\bar{k}, \bar{I}\}$. All else constant, increasing either firm's connectivity can benefit both firms' investment and profit.

All else equal, an increase in firm 1's connectivity makes its investment opportunity more sensitive to firm 2's investment decisions, and thus more sensitive to firm 2's liquidity shocks. As shown in Proposition 1, higher connectivity leads to a larger variation in investment, given the same range of liquidity realizations.

Proposition 1. *With higher connectivity ϕ_1, investment is more sensitive to a shock to total liquidity, i.e., the difference between the upper and lower limits of investment $\bar{k} - \underline{k}$ increases with ϕ_1.*

Proof for Proposition 1:

First, I show proof for the special case where $m_1 = m_2 = 0$. In this case, k_s/I_s is a constant, so λ_1 and λ_2 are also constants. There will be only one state. If $\lambda_1 > R$, firm 1 will always be in bank-borrowing state; if $\lambda_1 < 1-\delta$, firm 1 will always be in unconstrained state. Outside of these special cases, $1-\delta < \lambda < R$, i.e. firm 1 will be in inter-firm financing state. In this state, k_1 is proportional to total wealth. Denote $x = \frac{k_1}{I_1}$, which satisfies:

$$F = \alpha\phi_1^{1-\alpha}x^{\alpha-1} - (1-\alpha)\phi_1^{1-\alpha}x^{\alpha} - \alpha\phi_2^{1-\alpha}x^{1-\alpha} - (1-\alpha)\phi_2^{1-\alpha}x^{-\alpha} \tag{8}$$

It is easy to see that $\frac{\partial x}{\partial \phi_1} = \frac{\partial k_1/I_1}{\partial \phi_1} > 0$. Therefore, the proposition holds that k_1 is more sensitive to W when ϕ_1 is higher, i.e. with higher connectivity, a firm's time 1 investment is more sensitive to total liquidity.

The proof for a more general case where $m_1 m_2 \neq 0$ can be demonstrated through the following lemmas.

Lemma 1. *In the bank-borrowing states, the firms' internal value of liquidity equals bank lending rate R. The optimal second period investments of $\{k_{1,s}, I_{1,s}\}$ are fixed at a low level.*

Proof: Given the first-order conditions $\alpha(\frac{k_{1,s}}{\Phi_1})^{\alpha-1} = R$ and $\alpha(\frac{I_{1,s}}{\Phi_2})^{\alpha-1} = R$, we can solve for $\{k_{1,s}, I_{1,s}\}$. Denoting $x = (\frac{R}{\alpha})^{\frac{1}{\alpha-1}} = (\frac{\alpha}{R})^{\frac{1}{1-\alpha}}$, we have $\underline{k} = \frac{m_1 x + \phi_1 m_2 x^2}{1 - x^2 \phi_1 \phi_2}$.

Lemma 2. *In the inter-firm financing states, the optimal second period investments of both firms $\{k_{1,s},\ I_{1,s}\}$ only depend on total liquidity W_s, and strictly increase with total liquidity.*

Proof: I only need to show the proof for $k_{1,s}$, the other side is symmetric.

Recall the equilibrium condition is

$$F^c = (1-\alpha)(\frac{k_1}{\Phi_1})^{\alpha}\phi_1 - \lambda_1 - (1-\alpha)(\frac{I_1}{\Phi_2})^{\alpha}\phi_2 + \lambda_2 \tag{9}$$

In the inter-firm financing stage,

$\frac{\partial F^c}{\partial W_s} = \alpha(1-\alpha)k_{1,s}^{\alpha-1}\phi_1\Phi_1^{-\alpha}(W_s - k_{1,s}) + \alpha(1-\alpha)(W_s - k_{1,s})^{\alpha-2}\Phi_1^{-\alpha-1}(k_{1,s})\phi_2^2 + \alpha(1-\alpha)(W_s - k_{1,s})^{\alpha-2}\phi_2\Phi_2^{1-\alpha}(k_{1,s}) + \alpha(1-\alpha)(W_s - k_{1,s})^{\alpha-1}\phi_2\Phi_2^{-\alpha}(k_{1,s}) > 0$

Similarly, I can show that $\frac{\partial I}{\partial W_s} > 0$ in the constrained state. Q.E.D.

Lemma 3. *In the unconstrained states, i.e. when the realization of liquidity is high, firm 1 investment $k_{1,s}$ increases with firm 1's connection ϕ_1 and firm 2 investment $I_{1,s}$ increases with firm 2's connection ϕ_2.*

Proof: First define $F^u = \lambda_{i,s} - (1-\delta)$. This can be seen from:

$\frac{\partial F^u}{\partial \phi_1} = \alpha(1-\alpha)k_{1,s}^{\alpha-1}I_{1,s}\Phi_1^{-\alpha}(I_{1,s}) > 0$ Denoting $y = (\frac{1-\delta}{\alpha})^{\frac{1}{\alpha-1}} = (\frac{\alpha}{1-\delta})^{\frac{1}{1-\alpha}}$, we can solve for the investment level for firm 1: $\bar{k} = \frac{m_1 y + \phi_1 m_2 y^2}{1 - y^2\phi_1\phi_2}$, which increases with ϕ_1.

Q.E.D.

Comparing from the solution from Lemma 1 and Lemma 3, I show that the upper limit and the lower limit of firm 1 investment satisfy the following:

$$\bar{k} - \underline{k} = (y - x)\frac{m_1 + \phi_1(m_2(y+x) - m_1\phi_2 xy)}{(1 - x^2\phi_1\phi_2)(1 - y^2\phi_1\phi_2)}$$

Note that $y > x$, the numerator increases with ϕ_1 and the denomenator decreases with ϕ_1. Therefore, $\bar{k} - \underline{k}$ increases with ϕ_1.

Q.E.D.

IA 1.2.2 Liquidity

I illustrate through examples how firms' value of liquidity at $t = 1$ changes with economic states and connectivity.

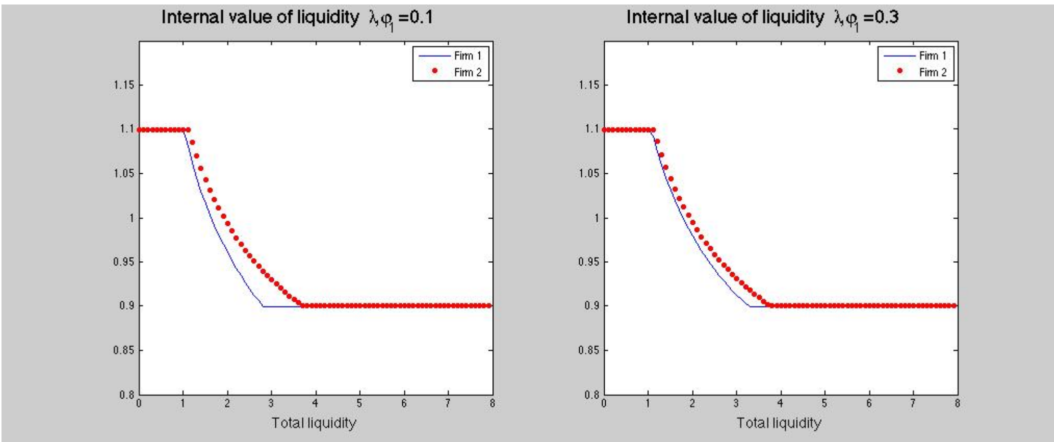


Figure IA1. Firm marginal value of liquidity
This figure shows the firms' value of internal liquidity as a function of total internal liquidity W_s. Left panel, $\phi_1 = 0.3$. Right panel, $\phi_1 = 0.1$. Parameter values: $R = 1.1$, $\alpha = 0.8$, $m_1 = 2$, $m_2 = 3$, $\phi_2 = 0.5$, $\delta = 0.1$.

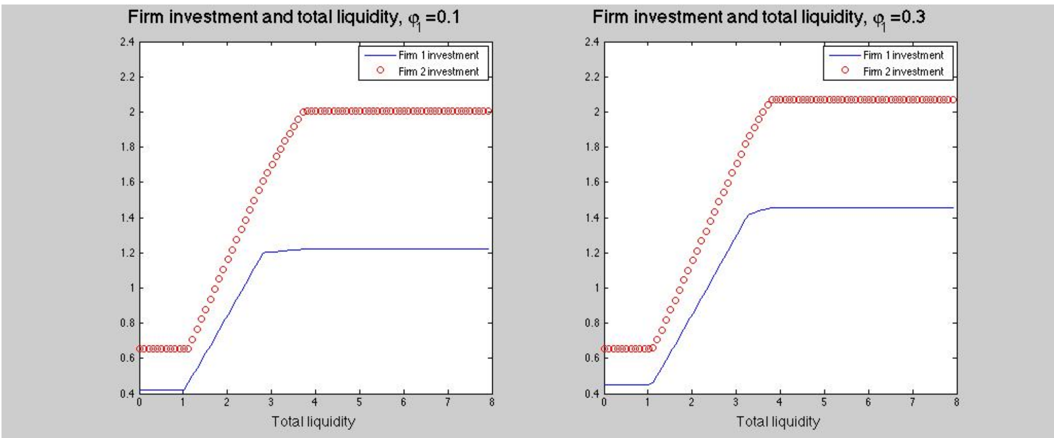


Figure IA2. Firm investment decisions
This figure shows the simulated investment of two firms as a function of total internal liquidity W_s. Left panel, $\phi_1 = 0.3$. Right panel, $\phi_1 = 0.1$. Parameter values: $R = 1.1$, $\alpha = 0.8$, $m_1 = 2$, $m_2 = 3$, $\phi_2 = 0.5$, $\delta = 0.1$.

Figure IA1 shows firms' marginal values of liquidity as functions of total liquidity W_s. When the realization of total liquidity W_s is very low, firms borrow from the bank and their marginal value of liquidity equals the bank rate. As W_s increases, firms start financing each other until they both reach the unconstrained state, when the value of liquidity equals the value of cash holdings inside firms. In the left panel, I set Firm 1's connectivity as $\phi_1 = 0.1$, and in the right panel, I increase connectivity to $\phi_1 = 0.3$. In both panels, Firm 2 connectivity is set to 0.5. The solid line shows Firm 1's value of liquidity and the dotted line shows Firm 2's value. The horizontal axis shows the total liquidity in the system $W_{1,s} + W_{2,s}$.

Several patterns emerge from this example. First, both firms assign a lower value to liquidity when they are collectively less constrained, i.e., when total liquidity is higher. Second, the firm with higher connectivity (Firm 2) values liquidity more than the other firm. Finally, comparing the left and the right panels, Firm 1 values liquidity more when it is more reliant on Firm 2, i.e., has a higher connectivity.

Figure IA2 shows firms' investments as functions of total liquidity W_s. Consistent with Proposition 1, firms' investment at very low realizations of liquidity is limited from below, because the bank rate is the highest cost of financing. In the inter-firm financing stage, investment increases with W_s until it reaches the upper bound at the unconstrained stage. Comparing the two panels, firm 1 investment exhibits a higher variation as firm 1 has a higher level of connectivity. In this case, firm 1's investment opportunity is more sensitive to firm 2's decisions.

Anticipating the cyclical investment patterns at $t = 1$, more connected firms should hold more cash at $t = 0$ to hedge against future liquidity shortage. To the extent that first period investment k_0 contributes to the cyclicality at $t = 1$, more connected firms also have an incentive to reduce this investment. In short, C_0 increases with ϕ, and k_0 decreases with ϕ.

IA 1.2.3 Cash Holdings

At $t = 0$, firms anticipate the future realization of liquidity and decide their initial cash holdings based on the first-order condition:

$$C_0 : \lambda_0 = (1 - \delta)^2 E[\lambda_s] \tag{10}$$

The left hand side indicates the marginal cost of holding cash (forgone immediate dividend payout), and the right hand side indicates the marginal benefit of holding cash (future investment opportunities multiplied by storage costs). As long as the marginal value of investment, λ_s, increases with connectivity ϕ_1, the marginal benefit of holding cash increases with connectivity. I provide the proof for cases $m_i = 0, \ i = 1, 2$.

Proposition 2. *If $m_i = 0, \ i = 1, 2.$, firms hold more cash when connectivity increases.*

Proof of Proposition 2:

Given that $\lambda_s = \alpha k_s^{\alpha-1} \mu_s$ and that $\lambda_0 \geq 1$ based on the first-order condition of P, we have:

$$C_0 = W_0 \quad if \quad (1 - \delta)^2 E[\alpha k_s^{\alpha-1} \mu_s] \geq 1 \tag{11}$$

$$C_0 = 0 \quad if \quad (1 - \delta)^2 E[\alpha k_s^{\alpha-1} \mu_s] < 1 \tag{12}$$

Given that $m_i = 0$, k_s/I_s is a constant that increases with ϕ_1. Specifically, $k_s/I_s = \sqrt{\frac{\phi_1}{\phi_2}}$. It follows that λ_s is a constant that increases with ϕ_1. Firms are more likely to hold cash $C_0 = W_0$ when they have high connectivity. Q.E.D.

IA 2 Pre-event Analyses for 9/11 Terrorist Attacks

In this section, I examine whether the trade credit usage of the firms in the event study of 9/11 attacks exhibited any pre-event trend. Specifically, I divide the pre-event window into two periods, with the first period being 3 to 4 years prior to the attacks and the second period being 1 to 2 years prior to the attacks. The post-event window can also be evenly divided into two-year periods. I then regress trade credit variables on the interaction of *Supplier* and dummy variables indicating each period while controlling for firm-fixed effects and year-fixed effects. The earliest pre-event window (year $[-4, -3]$) is absorbed as the base period.

Table IA1 shows the results from this test. Column (1) shows the effects for total receivables and Column (3) shows the results on payables for firms that had low cash holdings prior to the 9/11 attacks. This corresponds to the results presented in Column (2), Table 9 of the paper that only suppliers that had low cash holdings prior to the event increased their trade credit demand to upstream firms.

Overall, I find no significant difference between the trade credit of firms in the treated and control group prior to the event. The increases of receivables and net receivables emerged shortly following the 9/11 attacks, while accounts payable increased in a later time window. This effect reflects a delay in the transmission of shocks along the supply chain.

Table IA1 Pre-trend analysis: 9/11 terrorist attacks and trade credit usage
This table shows the changes in trade credit usage for airlines' suppliers prior to the 9/11 terrorist attacks. Column (1) shows results for receivables and Column (2) shows results for payables for the sample of firms that had bottom-tercile excess cash holdings during the pre-event window. Standard errors are clustered by firm.

	(1)	(2)
Sample	All	Low Cash
Dep. Var.:	*Receivables*	*Payables*
*Supplier*Event* $[-2, -1]$	0.004	0.013
	(0.40)	(0.82)
*Supplier*Event* $[0, 2]$	0.027**	0.023
	(2.58)	(1.62)
*Supplier*Event* $[3, 4]$	0.022	0.042**
	(1.46)	(2.44)
Firm FE	Yes	Yes
Year FE	Yes	Yes
Observations	7,264	2,520
R-squared	0.870	0.862

IA 3 Do Central Firms Alleviate Shock Transmission? Evidence from Natural Disasters

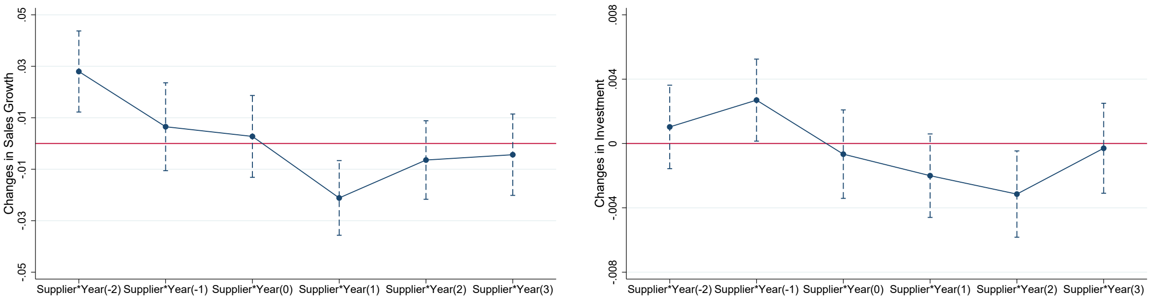
To the extent that central firms can choose conservative financial policies to preempt future shocks, they should be able to alleviate the negative impact of shocks that propagate in the production network. I design a test to validate this point. I exploit severe natural disasters as exogenous shocks whose negative impacts can spill-over to downstream firms in the network (Barrot and Sauvagnat (2016)). In other words, firms whose suppliers are affected by a natural disaster should experience a decline in performance in the years following the disaster. If central firms play a role in mitigating shock propagation, one should expect the decline in performance to be attenuated in cases where the suppliers facing the disaster have higher centrality, compared to the cases where the suppliers have low centrality.

I collect data from the Federal Emergency Management Agency (FEMA) and define severe disasters as those that last for at least 90 days. For each natural disaster, FEMA provides the counties that are affected by the disaster, which can be matched to firms' headquarter locations. I define treated firms as ones whose suppliers are headquartered in counties that have experienced a severe natural disaster. In other words, I define an indicator variable *SupplierDisaster* that equals one if a firm has at least one pre-established supplier that experiences a natural disaster in the past three years. Information on supplier-customer linkages comes from Compustat Segment database. For each natural disaster, I identify treatment based on supply-chain linkages that are already established in the year prior to the disaster. My control group consists of firms that are reported as customers but are not affected by natural disasters directly or through their suppliers.

First, I verify that treated firms experience deteriorating performance following natural disasters. To do so, I use both sales growth (defined as annual changes in the log of total sales) and investment as measures of firm performance and regress them on different event windows of the disaster. Figure IA3 shows the coefficient estimates around natural disasters. Both the sales growth and investment of affected firms decline significantly in the two-year window following natural disasters, but not prior to those disasters. This pattern validates the claim that natural disasters are negative, exogenous shocks that generate spillover effects along the supply chain.

Next, I examine whether central firms can mitigate the spillover impact of negative shocks. I define *Supplier Centrality* as the average centrality of suppliers of a firm in a given year, and divide it according to sample terciles. I then regress firm sales growth and investment on the interaction between terciles of *Supplier Centrality* with the occurrence of natural disasters to suppliers, *Supplier Disaster*. The estimation controls for firm-fixed effects (α_i) and year-fixed effects (η_t). It also controls for firm characteristics such as size, age, tangibility, and whether the firm is a dividend payer. Standard errors are clustered by firm.

Table IA2 reports the results. *Supplier Disaster* generates negative coefficients for both sales growth



(A) Sales Growth (B) Investment

Figure IA3. This figure shows the effect of natural disasters on downstream firms' sales growth and investment. *Suppliear*Year(τ)* is an indicator variable that equals one if a firm has a pre-established supplier that faces a natural disaster in its headquarter county in τ years prior to the current year of observation. For example, *Suppliear*Year(−2)* indicates 2 years prior to the disaster, and *Suppliear*Year(2)* indicates 2 years after the disaster. Panel (A) shows the effects for sales growth and Panel (B) shows the effects for investment. In each panel, the dots show coefficient estimates and the dashed lines indicate 90% confidence intervals. The estimation controls for firm-fixed effects and year-fixed effects. Standard errors are clustered by firm.

and investment, confirming that disaster shocks have negative implications for downstream firms. Importantly, the interaction between *High Supplier Centrality* and *Supplier Disaster* bears a positive, significant coefficient, suggesting that the negative spillover effect from disasters are mitigated when a firm is connected to a central supplier, compared to other firms that are connected to a non-central supplier. These results lend support to the argument that the conservative policies of central firms can help alleviate the spillover effect of negative shocks that propagate in the production network.

Granted, the above results from the natural disaster analysis suggest that downstream firms of central firms are more resilient to negative shocks. One may infer from such findings that central firms play a role in mitigating the spillover effect of negative shocks. Yet, I do not make causal statement regarding central firms' financial policy choices, since exogenous shocks to financial policies are rare and are often difficult to square with firms' optimizing their expected value.

Table IA2 Centrality and the spillover of shocks: Evidence from natural disasters

This table shows the changes in firms' sales growth and investment when a supplier is hit by a natural disaster. Column (1) shows results for sales growth and Column (2) shows results for investment. *Supplier Disaster* is an indicator for whether a firm's supplier has experienced a natural disaster in the past three years. *High (Medium) Supplier Centrality* is an indicator for whether a firm's suppliers have an average centrality that is in the top (middle) tercile of the sample. Standard errors are clustered by firm.

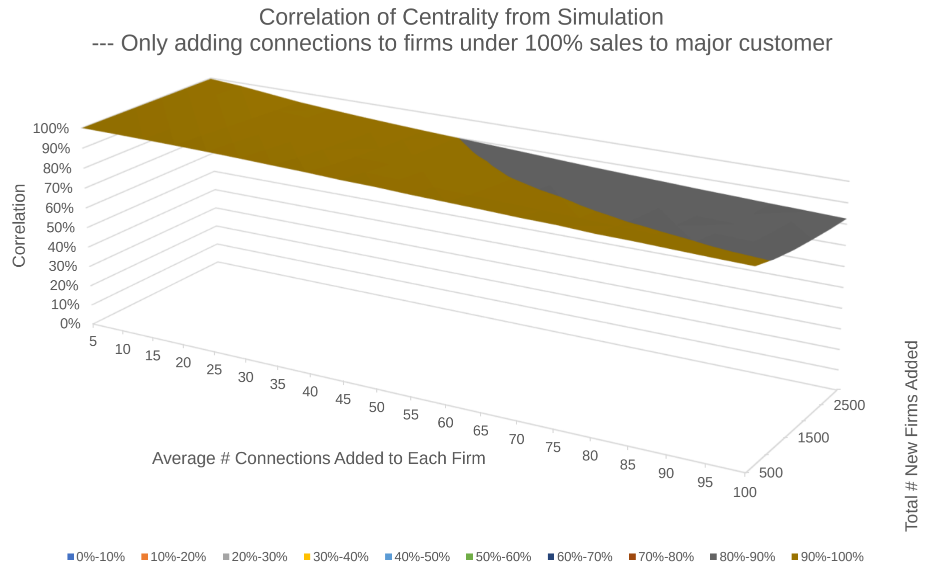
	(1)	(2)
Dep. Var.:	*Sales Growth*	*Investment*
Supplier Disaster	-0.015	-0.004*
	(-1.16)	(-1.71)
*Medium Supplier Centrality*Supplier Disaster*	-0.007	0.004
	(-0.47)	(1.36)
*High Supplier Centrality*Supplier Disaster*	0.037**	0.008**
	(2.17)	(2.27)
Firm Controls	Yes	Yes
Terciles of *Supplier Centrality*	Yes	Yes
Firm FE	Yes	Yes
Year FE	Yes	Yes
Observations	6,272	6,276
R-squared	0.359	0.723

IA 4 Addressing Concerns with *Closeness*

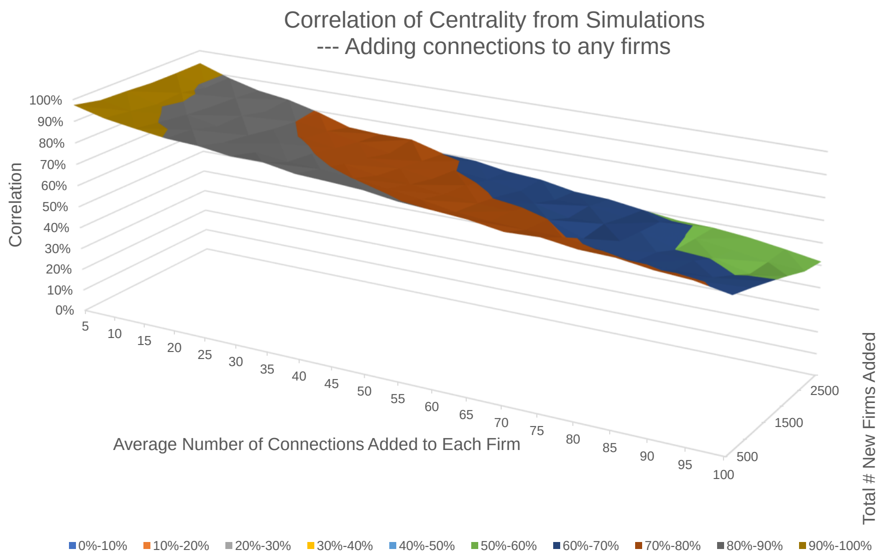
The Compustat segment database does not report customers that contribute less than 10% of a firm's total sales or customers of private firms. This truncation would lead to concerns that such a sample may introduce biases in the measurement of closeness centrality. I conduct simulations to assess the extent to which the Compustat sample could lead to measurement errors in *Closeness*.

I start with the observed network from Compustat in 2000 and add simulation-generated firms. I then randomly generate connections both between the existing and simulated firms, and among simulated firms themselves. In this augmented network, I compute the correlation of *Closeness* for the same firms in the extended network and in the original network.

Figure IA4 reports the correlation of a firm's *Closeness* in the original network and the augmented network. Panel (a) shows the correlation of *Closeness* when I add connections between simulated firms to existing firms who report sales to major customer below 100% (i.e., only simulating unobserved customers). Panel (b) removes this restriction, and adds new connections between any simulated firms and any existing ones. The high correlation reported in the figure suggests that, despite the increasing size of unobserved linkages, the structure of the observed network remains similar to the "true" networks. The simulation exercises thus suggest that the observed supply-chain network structure is robust to the omission of minor linkages.



(a)



(b)

Figure IA4. This figure demonstrates the robustness of closeness centrality for the observed firms in Compustat. I simulate networks with nodes and connections that are randomly added to the observed network based on the Compustat database. Panel (a) shows the correlation of closeness centrality between the original network and the simulation-augmented network. The latter is constructed by adding new firms and adding connections between the new firms and existing firms (reported by Compustat). In this panel, I only add connections to existing firms that report less than 100% their total sales to major customers. Panel (b) removes this restriction and randomly add connections between any new firms to any existing firms. In both panels, the x-axis (width) shows the average number of simulated connections added to each Compustat firm in each simulation, the y-axis (depth) shows the total number of new firms added in the augmented network, and the z-axis (height) shows the correlation of the closeness centrality in the simulation-augmented network and in the original network.